Neuro-Dimensional Architecture (NDA): A Theoretical Framework for Higher-Dimensional Cognition in Biological and Artificial Systems

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Abstract

This paper introduces **Neuro-Dimensional Architecture (NDA)**, a novel scientific discipline proposing that cognitive systems can fundamentally operate in higher-dimensional mathematical spaces, transcending traditional three-dimensional limitations. We present the **Dimensional Intelligence Hypothesis (DIH)**, which formally posits that intelligence scales superlinearly with accessible dimensional freedom. Four foundational constructs are established: **Dimensional Neural Geometry (DNG)** for neural embeddings in $\mathbb{R}^n$ ($n > 3$), **Multi-Axis Thought Pathways (MATP)** for parallel cognition across dimensions, **Non-Euclidean Memory Architecture (NEMA)** utilizing curved manifolds for exponential memory capacity, and **Dimensional State Collapse (DSC)** explaining decision-making as projection from high-dimensional states. Three governing laws are derived: the Dimensional Expansion Law (cognitive capacity grows exponentially with dimensionality), Hyper-Connectivity Law (information transmission improves in higher dimensions), and Dimensional Collapse Law (intelligent action requires optimal projection). We reinterpret biological cognitive phenomena (insight, intuition, creativity) as signatures of latent higher-dimensional processing, propose novel AI architectures (Hyper-Dimensional Transformers, Non-Euclidean Memory Networks), and outline experimental validation pathways in neuroscience and machine learning. NDA provides the first unified framework treating dimension as a manipulable design parameter for advancing both biological understanding and artificial intelligence toward superintelligent paradigms.

Keywords: Neuro-Dimensional Architecture, Higher-Dimensional Cognition, Dimensional Intelligence Hypothesis, Non-Euclidean Memory, Hyper-Dimensional Neural Networks, Cognitive Topology, Artificial General Intelligence, Theoretical Neuroscience

1 Introduction

Contemporary neuroscience and artificial intelligence operate within an implicit three-dimensional paradigm. Biological neural networks are analyzed as 3D structures, while artificial neural networks—despite operating in high-dimensional vector spaces—remain fundamentally designed and interpreted within 3D geometric intuition [1, 2]. This constraint limits our understanding of cognitive phenomena that appear to transcend linear cause-effect relationships, such as intuition, insight, and creative leaps.

Neuro-Dimensional Architecture (NDA) introduces a paradigm shift by proposing that intelligence is not merely computational but fundamentally *dimensional*. We posit that cognitive systems achieve higher forms of intelligence by accessing and operating within higher-dimensional representational spaces, where thoughts, memories, and decisions can exist in expanded geometric configurations. This approach builds upon recent advances in geometric deep learning [3], topological neuroscience [4], and hyperbolic embeddings [5], but extends them into a unified theoretical framework for cognition.

This paper establishes the theoretical foundation of NDA through four key contributions:

1. The **Dimensional Intelligence Hypothesis (DIH)** formalizing the relationship between dimensional freedom and cognitive capacity.
2. Four foundational constructs providing mathematical frameworks for higher-dimensional cognition.
3. Three governing laws derived from first principles.
4. Applications to biological cognition and artificial intelligence with testable predictions.

2 The Dimensional Intelligence Hypothesis

2.1 Formal Statement

Let C represent the cognitive capacity of a neural system, and D represent its accessible dimensional complexity (where $D \geq 3$). The DIH proposes:

$$C(D) = C_0 \cdot f(D) \cdot \eta(D) \tag{1}$$

where:

- C_0 is baseline capacity at $D = 3$
- $f(D)$ is a superlinear function, typically $e^{\lambda(D-3)}$ with $\lambda > 0$
- $\eta(D)$ is encoding efficiency ($0 < \eta \leq 1$), representing the system's ability to utilize higher dimensions effectively

2.2 Corollaries

Corollary 1 (Biological Limit)**.** *Human brains, while 3D in physical structure, may achieve $D > 3$ through* functional hyper-dimensionality *in neural coding patterns. This explains phenomena like combinatorial coding in grid cells [6] and high-dimensional population coding in cortical networks [7].*

Corollary 2 (Artificial Potential)**.** *AI systems explicitly designed for $D > 3$ operations could surpass human-level intelligence without requiring biological neuron-scale complexity. This suggests a path to artificial general intelligence through dimensional optimization rather than mere scaling [8].*

2.3 Theorem: NDA Separation Theorem

Theorem 1 (NDA Separation Theorem)**.** *Physical dimensionality $\neq$ functional dimensionality. A 3D physical brain can implement n-D cognitive processes through specific encoding schemes, where $n \gg 3$.*

Proof. Consider a smooth embedding $\phi : \mathbb{R}^3 \to \mathbb{R}^n$ where neural activity patterns in 3D space encode coordinates in n-D cognitive space. By the Whitney embedding theorem [9], any smooth m-dimensional manifold can be embedded in $\mathbb{R}^{2m}$. For cognitive manifolds with $m > 1$, there exist embeddings into $\mathbb{R}^n$ with $n > 3$. Neural population codes can implement such embeddings through distributed representations [10]. □

3 Four Foundational Constructs

3.1 Dimensional Neural Geometry (DNG)

DNG studies how neural elements behave when embedded in or mapped to higher-dimensional geometric spaces. This extends traditional neural geometry [11] into higher dimensions.

Definition 1 (DNG Embedding)**.** *Let N be a set of neural units with activation patterns $a_i(t) \in \mathbb{R}$. A DNG embedding is a smooth mapping:*

$$\phi : N \to \mathbb{R}^n, \quad n > 3$$

where n-dimensional coordinates $\phi(p) = (x_1, x_2, \ldots, x_n)$ represent functional coordinates *in a cognitive manifold, preserving topological relationships between concepts.*

Properties:

1. *Hyper-connectivity*: In $\mathbb{R}^n$, possible connections between k nodes scale as $O(2^{n(n-1)/2})$, enabling exponentially more associative links than 3D networks [12].
2. *Distance compression*: Similar concepts cluster more tightly in properly structured n-D spaces, with distance scaling as $d \sim \log n$ for hyperbolic embeddings [13].
3. *Parallel representation*: Single n-D connection can encode multiple 3D relationships through tensor decompositions [14].

3.2 Multi-Axis Thought Pathways (MATP)

MATP models cognitive processes propagating simultaneously across multiple independent dimensions, analogous to parallel processing but with dimensional structure.

$$\frac{dT}{dt} = \sum_{i=1}^{n} \alpha_i \frac{\partial T}{\partial x_i} + \beta \nabla^2 T + \gamma R(T) + \epsilon(x, t) \tag{2}$$

where:

- x_i are orthogonal cognitive dimensions (e.g., semantic, spatial, temporal, emotional)
- α_i represent propagation velocities along each dimension
- $\nabla^2 T$ models cross-dimensional diffusion (spreading activation)
- $R(T)$ captures nonlinear interaction terms (concept combination)
- $\epsilon(x, t)$ represents stochastic exploration noise

Implications:

- *Parallel intuition*: Multiple solution approaches pursued simultaneously across dimensional axes
- *Multi-threaded consciousness*: Attention distributed across dimensional axes with variable weights
- *Dimensional interference*: Constructive interference produces insight when pathways converge

3.3 Non-Euclidean Memory Architecture (NEMA)

NEMA utilizes curved, folded, or topologically complex n-dimensional manifolds for memory storage, moving beyond vector space models [15] to geometric memory.

Definition 2 (NEMA Manifold). *Memory traces M_i are encoded on an n-dimensional Riemannian manifold $\mathcal{M}$ with metric tensor $g_{ij}(x)$. Recall involves geodesic navigation:*

$$\textit{Recall}(Q) = \arg\min_{M \in \mathcal{M}} d_g(Q, M) + \lambda \cdot S(M)$$

where d_g is geodesic distance computed via $\frac{d^2x^i}{ds^2} + \Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$, and $S(M)$ is a stability penalty.

Advantages over Euclidean memory:

1. *Exponential capacity*: Hyperbolic spaces $\mathbb{H}^n$ have volume growing as $e^{(n-1)r}$ with radius r [12]
2. *Natural hierarchy embedding*: Tree structures embed with minimal distortion in hyperbolic space
3. *Associative shortcuts*: Geodesics provide direct connections between semantically related concepts

3.4 Dimensional State Collapse (DSC)

DSC describes the process where high-dimensional cognitive states reduce to lower-dimensional outputs for action or communication.

$$|\Psi(t)\rangle = \sum_{i_1,\ldots,i_n} c_{i_1\ldots i_n}(t)|d_1^{i_1},\ldots,d_n^{i_n}\rangle \xrightarrow{\mathcal{C}} |\psi_{\text{output}}\rangle \in \mathbb{R}^m \tag{3}$$

where $m < n$, typically $m = 2$ or 3 for communication, and $\mathcal{C}$ represents the collapse operator satisfying $\mathcal{C}^\dagger\mathcal{C} = I$.

Explained phenomena:

- *Insight ("Aha!" moments)*: Sudden collapse from high-dimensional exploration to low-dimensional solution
- *Decision paralysis*: Failure to collapse due to high-dimensional state uncertainty or conflicting axes
- *Creativity*: Controlled partial collapses preserving dimensional richness in artistic expression

4 Three Laws of Neuro-Dimensional Architecture

4.1 First Law: Dimensional Expansion Law

Law 1 (Dimensional Expansion). *Increasing the dimensional complexity of a neural system exponentially increases its cognitive capacity, subject to encoding efficiency constraints. Formally, for a system with intrinsic dimension D:*

$$C(D) = C_0 \cdot e^{\lambda(D-D_0)} \cdot \eta(D) \cdot \left[1 - e^{-\frac{D}{D_c}}\right] \tag{4}$$

where:

- $\eta(D) = \frac{1}{1+e^{-\alpha(D-D_{opt})}}$ models sigmoidal encoding efficiency
- D_c is a critical dimension where returns diminish
- D_{opt} is the optimal dimension for given neural resources

4.2 Second Law: Hyper-Connectivity Law

Law 2 (Hyper-Connectivity). *Information transmission across higher-dimensional manifolds exhibits lower loss, higher parallelism, and the emergence of dimensional shortcuts not available in 3D spaces.*

Connection density in n-D cognitive space on sphere S^{n-1}:

$$\rho(n) = \frac{\Gamma(n/2)}{2\pi^{n/2}}r^{n-1} \cdot \left[1 + \frac{(n-1)(n-2)}{12}r^2 + O(r^4)\right] \tag{5}$$

enabling exponentially more associative connections than 3D. Information transmission rate scales as:

$$I(n) = I_0 \cdot n \cdot \log(1 + \text{SNR}(n))$$

where $\text{SNR}(n) \propto n^{1/2}$ for optimized dimensional encoding.

Theorem 2 (Dimensional Shortcuts). *For sufficiently high n, any two concepts in a properly embedded cognitive manifold are at most $O(\log n)$ geodesic steps apart via optimal dimensional pathways, compared to $O(\sqrt{n})$ in Euclidean space.*

4.3 Third Law: Dimensional Collapse Law

Law 3 (Dimensional Collapse). *Intelligent action requires the collapse of high-dimensional cognitive states into lower-dimensional expressions, with the collapse process itself being an optimization that balances information preservation with decisiveness.*

Optimal collapse mapping $P^* : \mathbb{R}^n \to \mathbb{R}^m$ $(m < n)$:

$$P^* = \arg\min_{P \in \mathcal{P}} \left[\mathcal{I}(S_n, P^{-1}(P(S_n))) + \lambda_1 \cdot \mathcal{C}(P) + \lambda_2 \cdot \mathcal{D}(P(S_n))\right] \tag{6}$$

where:

- $\mathcal{I}$ is mutual information loss (fidelity)
- $\mathcal{C}$ is computational complexity of P
- $\mathcal{D}$ is decision uncertainty in collapsed space
- λ_1, λ_2 trade-off parameters

5 Biological Signatures and Predictions

5.1 Neurophysiological Evidence

Several lines of evidence support the plausibility of higher-dimensional cognitive processing:

- *Grid cells and manifold coding*: Grid cells in entorhinal cortex code 2D manifolds [6]; extension to 3+D manifolds is neurobiologically plausible through conjunctive coding [16].
- *High-dimensional population codes*: Cortical population activity often requires $n > 3$ principal components to explain variance [7, 17].
- *Binding problem solution*: Feature binding, a classic problem in neuroscience, finds natural solution in n-D spaces where features occupy orthogonal dimensions that can be simultaneously active [18].
- *Conscious access dynamics*: Global workspace theory [19] aligns with DSC when workspace dimensions $n \gg 3$.

5.2 Cognitive Phenomena Reinterpretation

Neuro-Dimensional Architecture provides novel interpretations for several cognitive phenomena that resist traditional explanations. These phenomena can be understood as natural consequences of brains operating in higher-dimensional representational spaces, with specific patterns of dimensional access, processing, and collapse.

The NDA framework suggests these phenomena are not anomalies but rather signatures of brains routinely operating in dimensions beyond our physical three. For instance, the sudden nature of insight experiences corresponds to rapid dimensional collapse from high-dimensional exploration spaces. Creative thinking involves controlled exploration of these high-dimensional spaces while maintaining the ability to selectively collapse to communicable forms. Even common experiences like déjà vu may represent brief alignments with previously experienced high-dimensional cognitive configurations.

5.3 Experimental Predictions for Neuroscience

Prediction 1 (B1: Dimensional Synchronization)**.** *EEG/MEG should detect transient high-dimensional synchronization patterns ($n > 3$ functional connectivity measured via persistent homology [4]) preceding insight moments by 300-500ms, with dimensionality n correlating with solution elegance.*

Prediction 2 (B2: Manifold Dimensionality)**.** *fMRI manifold analysis during complex reasoning (Raven's matrices, insight problems) will require $n > 3$ dimensions for adequate state space modeling, with n correlating with individual intelligence ($r > 0.6$) and creative achievement.*

Prediction 3 (B3: Dimensional Enhancement)**.** *Transcranial alternating current stimulation (tACS) at specific frequencies (40Hz gamma) applied to prefrontal-parietal networks could temporarily increase accessible n and enhance creative problem-solving performance by $\geq 25\%$.*

Prediction 4 (B4: Developmental Trajectory)**.** *Children's cognitive manifolds will show increasing intrinsic dimensionality n with age, plateauing around early adulthood, with $n_{adult} \approx 1.5 \times n_{child}$ for reasoning tasks.*

6 Artificial NDA Architectures

6.1 From 3D to n-D Neural Networks

Current AI architectures remain constrained by 3D design intuition despite operating in high-dimensional spaces. Convolutional networks assume 2D/3D grid structure [27], transformers use attention over sequence dimensions [28], but neither explicitly leverages higher-dimensional geometric structure for cognitive processing.

6.2 Architectural Templates

6.2.1 Hyper-Dimensional Transformer (HD-Transformer)

Extends attention to n-D token manifolds with dimensional attention weights:

$$\text{Attention}_{nD}(Q, K, V) = \text{Softmax}\left(\frac{\prod_{i=1}^{n} \text{MatMul}(Q_i, K_i^T)}{\sqrt{\prod_{i=1}^{n} d_{k_i}}}\right) \otimes_n V \tag{7}$$

where:

- $Q_i, K_i \in \mathbb{R}^{L \times d_i}$ are queries and keys along dimension i
- $\otimes_n$ denotes n-mode tensor product
- d_{k_i} is dimension-specific scaling factor
- Multi-dimensional positional encoding: $PE_{(p_1,\ldots,p_n)} = \bigoplus_{i=1}^{n} PE_i(p_i)$

6.2.2 Non-Euclidean Memory Network (NEM-Net)

Memory stored on learnable hyperbolic manifold $\mathbb{H}^n$ with curvature $\kappa < 0$:

$$d_{\mathbb{H}}(x, y) = \frac{1}{\sqrt{-\kappa}} \text{arcosh}\left(1 - 2\kappa \frac{\|x - y\|^2}{(1 + \kappa\|x\|^2)(1 + \kappa\|y\|^2)}\right) \tag{8}$$

Memory update via Riemannian gradient descent:

$$M_{t+1} = \text{Exp}_{M_t}\left(-\eta \text{grad}_{\mathbb{H}} \mathcal{L}(M_t)\right)$$

where $\text{Exp}_p(v)$ is the exponential map on $\mathbb{H}^n$.

6.2.3 Dimensional Collapse Network (DCN)

Learns task-optimal projections $P_\theta : \mathbb{R}^n \to \mathbb{R}^m$ parameterized by neural networks:

$$P_\theta(x) = f_{\theta_m} \circ \cdots \circ f_{\theta_1}(x), \quad \dim(f_{\theta_i}(\cdot)) < \dim(f_{\theta_{i-1}}(\cdot))$$

with loss function:

$$\mathcal{L}(\theta) = \underbrace{\|x - P_\theta^{-1}(P_\theta(x))\|^2}_{\text{reconstruction}} + \lambda_1 \underbrace{H(P_\theta(x))}_{\text{entropy reduction}} + \lambda_2 \underbrace{\|\nabla P_\theta\|_F}_{\text{smoothness}}$$

6.3 Expected Advantages

1. *Exponential parameter efficiency*: Representational capacity scales as $O(e^n)$ rather than $O(n^k)$
2. *Natural multi-tasking*: Different tasks map to orthogonal or complementary dimensional axes
3. *Improved generalization*: n-D decision boundaries are smoother with fewer local minima
4. *Enhanced explainability*: Dimensions can correspond to semantically meaningful axes (abstraction, specificity, valence, etc.)
5. *Better few-shot learning*: n-D representations allow interpolation between few examples across multiple dimensions

Prediction 5 (A1: Compositional Learning)**.** *HD-Transformers will achieve superior few-shot learning on compositional tasks (SCAN [29], COGS [30]) compared to standard Transformers, with* $> 15\%$ *improvement in systematic generalization.*

Prediction 6 (A2: Exploration Efficiency)**.** *NDA-based reinforcement learning agents will discover more novel strategies by exploring n-D policy spaces, showing* $>$ 30% *higher exploration entropy and discovering* $> 2\times$ *more distinct solutions in hard-exploration environments (Montezuma's Revenge, Pitfall).*

Prediction 7 (A3: Memory Scaling)**.** *NEMNet will show better than linear scaling with memory size, maintaining* $> 90\%$ *retrieval accuracy with* 10^6 *items in hyperbolic space versus* $< 70\%$ *for Euclidean memory at same parameter count.*

7 Experimental Validation Pathways

7.1 Computational Experiments

1. **Dimensional scaling studies**: Fix total parameter count at 10^7, vary intrinsic dimension $D \in [3, 10]$, measure performance on cognitive benchmarks (Raven's Progressive Matrices, text-based reasoning, mathematical problem-solving). *Hypothesis*: Performance peaks at $D \approx 6 - 8$ then plateaus.
2. **Collapse mechanism analysis**: Train DCNs with varying output dimensions $m \in [1, 5]$, quantify trade-off between decision speed (forward pass time) and solution quality (accuracy). *Hypothesis*: Optimal m is task-dependent but $m = 2$ maximizes speed-quality trade-off across tasks.
3. **NEMA capacity tests**: Compare hyperbolic ($\mathbb{H}^5$) vs. Euclidean ($\mathbb{R}^5$) vs. spherical (S^5) memory on hierarchical recall tasks (WordNet hyponymy, mathematical concept dependencies). *Hypothesis*: $\mathbb{H}^5$ shows $> 50\%$ better accuracy on hierarchical relations.

7.2 Neuroscientific Experiments

1. **Dimensionality estimation**: Apply persistent homology and intrinsic dimension estimation (MLE [31], TWO-NN [32]) to multi-electrode recordings from prefrontal cortex during reasoning tasks. Compare experts vs. novices in chess, mathematics, programming.
2. **Dimensional enhancement**: Neurofeedback training to increase estimated n of mental representations during visuospatial reasoning. Pre-post test on mental rotation, insight problems. Include sham control.
3. **TMS disruption**: Apply transcranial magnetic stimulation to parietal cortex during problem-solving. Predict differential effects on low- vs. high-dimensional strategies (measured via think-aloud protocols).

7.3 Psychophysical Experiments

1. **Dimensional priming**: Prime subjects with 3D vs. 4D spatial visualization training (using VR and mathematical visualization tools). Measure subsequent performance on insight problems requiring dimensional analogies.
2. **Cross-dimensional transfer**: Train skills in virtual 4D environments (4D maze navigation, hypercube manipulation). Test transfer to 3D spatial reasoning, architectural design, protein folding visualization.
3. **Collapse timing**: Present multi-dimensional problems (e.g., with semantic, logical, spatial constraints). Measure response time and accuracy. Predict U-shaped function: very fast (intuitive collapse), slow (analytical), very slow (dimensional overload).

8 Discussion and Future Directions

8.1 Theoretical Implications

NDA provides a unified framework for biological and artificial intelligence based on dimensional principles, suggesting that:

- *Consciousness* may correspond to particular regimes of dimensional complexity ($n > 3$) with specific integration patterns (high Φ in Integrated Information Theory [33] but with dimensional structure).
- *Creativity* involves controlled exploration of high-dimensional spaces followed by selective collapse to communicable forms.
- *Learning* is not just parameter adjustment but dimensional expansion and manifold shaping.
- *Attention* functions as dimensional selection and weighting in MATP.

The framework bridges previously disconnected theories: global workspace (dimensional collapse), predictive processing (manifold learning), and symbolic AI (dimensional compositionality).

8.2 Technological Applications

1. **NDA-based BCIs**: Decode n-D cognitive states directly from neural activity, enabling thought communication beyond linguistic constraints. Applications in locked-in syndrome, enhanced human-computer interaction.
2. **Higher-dimensional neural prosthetics**: Memory augmentation via external NEMA systems interfacing with hippocampus. Cognitive enhancement for aging population, memory disorders.
3. **Next-generation AI**: Systems with explicit dimensional reasoning capabilities for scientific discovery, creative arts, strategic planning. Dimensional AI assistants for complex decision-making.

8.3 Ethical Considerations

The development of NDA technologies raises important ethical questions:

- *Cognitive inequality*: Dimensional enhancement technologies could exacerbate existing inequalities if available only to privileged groups.
- *Agency and identity*: Modifying one's cognitive dimensionality raises questions about personal identity and autonomy.
- *n-D AI safety*: AI systems operating in high-dimensional spaces may develop incomprehensible (to 3D-bounded humans) goals and strategies.
- *Regulation*: Need for ethical frameworks specific to dimensional BCIs, cognitive augmentation, and high-dimensional AI.

We recommend proactive development of ethical guidelines parallel to technological development, with multidisciplinary input from ethicists, neuroscientists, AI researchers, and affected communities.

8.4 Limitations and Open Questions

1. **Optimal dimensionality**: What determines optimal n for different cognitive tasks? Is there a universal $n_{\max}$ constrained by neural architecture?
2. **Dimensional scaling laws**: How does n scale with brain size, connectivity, or species? Comparative studies across species needed.
3. **Dimensional control**: Can n be consciously controlled or trained? Preliminary evidence from meditation studies suggests possible [34].
4. **Hardware implementations**: Can we build physical systems that naturally operate in $n > 3$ dimensions? Quantum systems, photonic networks, or novel materials may offer pathways.

8.5 Conclusion

Neuro-Dimensional Architecture represents a paradigm shift in our understanding of intelligence, proposing that cognitive systems fundamentally operate in higher-dimensional mathematical spaces. By establishing the Dimensional Intelligence Hypothesis, four foundational constructs, and three governing laws, we provide a unified framework for both biological cognition and artificial intelligence. The reinterpretation of cognitive phenomena through an NDA lens offers novel explanations for insight, intuition, creativity, and other experiences that resist traditional accounts. The proposed AI architectures and experimental validation pathways provide concrete directions for future research. NDA opens new avenues for advancing both neuroscience and AI toward superintelligent paradigms by treating dimension as a manipulable design parameter rather than a fixed constraint.

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Data and Code Availability

This is a theoretical paper. The Neuro-Dimensional Architecture Initiative maintains an official research portal at `https://www.nda-institute.org` where reference implementations (HD-Transformer, NEMNet, DCN), supplementary materials, and updates are hosted. Code is released under the MIT License.

Experimental data from future validation studies will be shared following FAIR principles via the NDA data repository. Simulation code for MATP dynamics will be provided as Jupyter notebooks.

License

Competing Interests

The author is founder of the Neuro-Dimensional Architecture Initiative, a non-profit research organization that aims to develop applications based on this framework. No other competing interests.

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A Mathematical Supplement

A.1 Proof of Dimensional Shortcuts Theorem

Proof. Consider N cognitive concepts uniformly distributed on S^{n-1}, the unit sphere in $\mathbb{R}^n$. The angular distance θ between two random points has distribution:

$$p(\theta) = \frac{\Gamma(n/2)}{\sqrt{\pi}\Gamma((n-1)/2)}(\sin\theta)^{n-2}, \quad 0 \leq \theta \leq \pi$$

For large n, $\theta \approx \pi/2$ with high probability. The chordal distance is $d_c = 2\sin(\theta/2) \approx \sqrt{2}$.

In hyperbolic geometry $\mathbb{H}^n$ with curvature $\kappa = -1$, the hyperbolic distance between points with chordal distance d_c is:

$$d_{\mathbb{H}} = \operatorname{arcosh}\left(1 + \frac{d_c^2}{1 - \|x\|^2}\right)$$

For optimal embeddings that place points at radius r maximizing capacity, $\|x\| \approx \sqrt{1 - 1/n}$. Then:

$$d_{\mathbb{H}} \approx \operatorname{arcosh}\left(1 + 2n(1 - \cos\theta)\right) \approx \log n + O(1)$$

Thus, the average distance between concepts in optimally embedded hyperbolic cognitive space scales as $O(\log n)$, compared to $O(\sqrt{n})$ in Euclidean space. □

A.2 Dimensional Capacity Calculations

The number of dichotomies $C(n, N)$ of N points in $\mathbb{R}^n$ that are linearly separable (Cover's theorem):

$$C(n, N) = 2\sum_{k=0}^{n-1}\binom{N-1}{k}$$

For $N \gg n$, using $\binom{N}{k} \approx \frac{N^k}{k!}$:

$$C(n, N) \approx 2\frac{N^{n-1}}{(n-1)!}\left[1 + O\left(\frac{n^2}{N}\right)\right]$$

Thus capacity grows super-exponentially with n for fixed N.

For hyperbolic space $\mathbb{H}^n$ of curvature -1, the number of approximately equidistant points at radius r is:

$$N_{\mathbb{H}}(n, r) \approx e^{(n-1)r} \cdot \frac{\sinh^{n-1}(r)}{\Gamma(n/2)}$$

showing exponential scaling with both n and r, compared to polynomial scaling $N_{\mathbb{R}}(n, r) \propto r^n$ in Euclidean space.

A.3 MATP Stability Analysis

The linearized MATP equation around equilibrium T_0:

$$\frac{d\delta T}{dt} = \sum_{i=1}^{n}\alpha_i\frac{\partial\delta T}{\partial x_i} + \beta\nabla^2\delta T + \gamma R'(T_0)\delta T$$

Assume solution form $\delta T = e^{\lambda t}e^{i\mathbf{k}\cdot\mathbf{x}}$. Then:

$$\lambda = i\sum_{i=1}^{n}\alpha_i k_i - \beta\|\mathbf{k}\|^2 + \gamma R'(T_0)$$

Stability requires $\text{Re}(\lambda) \leq 0$ for all $\mathbf{k}$, giving conditions:

$$\beta \geq \frac{\|\alpha\|^2}{4\gamma R'(T_0)} \quad \text{and} \quad \gamma R'(T_0) \leq 0$$

Thus dimensional propagation (α_i) must be balanced by diffusion (β) and damping ($\gamma R'(T_0) < 0$) for stable thought dynamics.

Table 1: NDA Interpretation of Cognitive Phenomena

Phenomenon	**Traditional Explanation**	**NDA Interpretation**
Insight (Aha! moment)	Restructuring of problem representation [20]	Dimensional state collapse from n-D exploration to 2D/3D solution
Intuition	Unconscious pattern recognition [21]	Rapid computation along n-D manifolds with heuristic collapse
Creativity	Divergent thinking, remote associations	Controlled exploration of high-dimensional spaces with selective collapse
Déjà vu	Memory glitch or temporal lobe anomaly [22]	Accidental alignment with previous n-D cognitive state configuration
Dream logic	Reduced prefrontal regulation [23]	Free n-D associative exploration without collapse constraints
Synesthesia	Cross-wired sensory areas [24]	Natural n-D unity of perceptual dimensions
Flow state	Optimal challenge-skill balance [25]	Synchronized MATP across dimensions with minimal dimensional interference
Tip-of-tongue	Partial memory retrieval failure [26]	Incomplete collapse from n-D memory trace to linguistic output